UUCMS. No.

B.M.S COLLEGE FOR WOMEN

BENGALURU – 560004

I SEMESTER END EXAMINATION – APRIL 2024

M.Sc. MATHEMATICS - ORDINARY DIFFERENTIAL EQUATIONS
(CBCS Scheme-F+R)

Course Code MM104T **QP Code: 11004**
Duration: 3 Hours **Max. Marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. (a) Establish Liouville's formula for n^{th} order differential equation in its usual form.
   (b) Find the Wronskian of independent solution of $y^5 - y^4 - y' - y = 0$, $x \in (-\infty, \infty)$. **(7+7)**

2. (a) State and prove Sturm's comparison theorem on the zeros of a self-adjoint differential equation.
   (b) Solve the differential equation $x^2y'' - xy' - 3y = x^3$ by the method of variation of parameters **(7+7)**

3. (a) Prove that the eigenfunctions corresponding to distinct eigenvalues of a self-adjoint eigenvalue problem are orthogonal over the relevant interval.
   (b) Find the eigenvalues and eigenfunctions of the differential equation
   $\frac{d}{dx}(xy') + \frac{\lambda}{x}y = 0;\ y'(1) = 0 = y'(e^{2\pi})$. **(7+7)**

4. (a) Find the general solution of $x^2y'' + 7xy' + 8y = 0$ by finding the solution of its adjoint equation.
   (b) Establish the Green's function for the problem
   $y'' + \lambda y = x, y(0) = 0 = y(\pi)$. **(7+7)**

5. (a) Discuss about ordinary and singular point of the equation $x\frac{d^2y}{dx^2} + (1 - x)y' + \alpha y = 0$.
   (b) Find a solution of the Hermite equation $y'' - 2xy' + 2\alpha y = 0$ using Frobenius method about the ordinary point x=0.
   (c) Prove that e^{2xt-t^2} is the generating function for the Hermite polynomial. **(4+5+5)**

6. (a) Derive the following recurrence relation for the Laguerre polynomials:
   $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$

(b) Prove that $\int_0^\infty e^{-x} L_m(x) L_n(x) dx = \begin{cases} 1, m = n \\ 1, m \neq n \end{cases}$. **(7+7)**

7. Find the fundamental matrix solution of the following system of equations

(a) $\frac{dx}{dt} = 6x - 3y;\ \frac{dy}{dt} = 2x + y$

(b) $\frac{dx}{dt} = x + y - 5t + 2;\ \frac{dy}{dt} = 4x - 2y - 8t - 8$ **(7+7)**

8. (a) Determine the nature and stability of the critical point of the system

$$\frac{dx}{dt} = 8x - y^2;\ \frac{dy}{dt} = -6y + 6x^2.$$

(b) Apply Liapunov direct method to determine the stability of the critical point (0, 0) of the system $\frac{dx}{dt} = -x^5 - y^3;\ \frac{dy}{dt} = 3x^3 - y^3$. **(7+7)**

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